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**Forecasting State Income Tax Receipts:  
A Time Series Approach**

*Nader Nazmi*

*Jane H. Leuthold*

College of Commerce and Business Administration  
Bureau of Economic and Business Research  
University of Illinois, Urbana-Champaign



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## Forecasting State Income Tax Receipts: A Time Series Approach

Nader Nazmi, Ph.D. Candidate  
Department of Economics

Jane H. Leuthold, Professor  
Department of Economics

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## ABSTRACT

In this study time series analysis is applied to the problem of forecasting state income tax receipts. An objective criterion developed by Hannon and Quinn (1979) is applied to identify the model and a Box-Cox (1964) transformation is used to select between the log and linear versions of the model. Out-of-sample forecasts from the model are compared to forecasts obtained from an econometric model. The time series model consistently outperforms the econometric model in forecasting state tax receipts according to the percentage root mean square error test. The study establishes time series analysis as a viable technique for forecasting state tax receipts.



## I. Introduction

The growing consequence of state income tax revenues makes their accurate forecasting important. The techniques currently used include both single equation econometric models such as those of Singer (1968) and Greytak and Thursby (1980) and simultaneous equation models such as one developed by Auten and Robb (1976). The data may be either quarterly or annual and explanatory variables often include income, population, income per capita, and the tax rate. The purpose of this paper is to demonstrate the application of another forecasting technique, time series analysis, to the problem of forecasting state tax revenues.

Time series analysis offers some advantages over econometric forecasting. It has the advantage of not requiring a large amount of data and it can handle seasonal fluctuations better than the regression technique. The disadvantage is that its application requires an experienced researcher to identify the model. Forecasting with a time series model is often more like an art than a science. We propose to reduce the subjectivity associated with time series analysis by employing an objective criterion for identifying the model. We also apply the Box-Cox (1964) transformation to discriminate between the linear and the log form of the model.

The data used in this study are for the state of Illinois but the techniques are readily applicable to that of other states. The data are quarterly Illinois income tax receipts for the period 1970 I through 1980 IV. The model is used to forecast over the period 1981 I through 1983 II.<sup>1</sup> The forecasts are evaluated using the percentage

root mean square error for the out-of-sample data and are compared to forecasts obtained from an econometric model relating income tax receipts to personal income. Our results show that a properly specified time series model outperforms a single equation econometric model according to the percentage mean square error loss function.

In Section II, the evaluation criterion used in the study is explained and justified. In Section III, the results of employing the evaluation criterion to select the "best" model are presented and in Section IV, the time series forecasts are compared with those obtained from an econometric forecasting model. In Section V, the results are evaluated.

## II. Time Series Methodology

As the first step in our investigation, we have to find the "true" order of the underlying linear and log linear ARMA (p,q) process:

$$X_t = \theta_0 + \theta_1 X_{t-1} + \dots + \theta_p X_{t-p} + \varepsilon_t + \delta_1 \varepsilon_{t-1} + \dots + \delta_q \varepsilon_{t-q}$$

$$\log X_t = \beta_0 + \beta_1 \log X_{t-1} + \dots + \beta_p \log X_{t-p} + \varepsilon_t + \gamma_1 \varepsilon_{t-1} + \dots + \gamma_q \varepsilon_{t-q}.$$

Following Box and Jenkins (1970), the conventional approach to the problem of model selection in time series suggests that the sample autocorrelation and partial autocorrelation functions should be employed for determining the order of the generating process. Of course, this procedure assumes that the mentioned sample statistics closely resemble the autocorrelation and partial autocorrelation functions of the unknown process. Furthermore, this procedure requires a skillful researcher who, equipped with vast experience, can determine

the order of the process with visual inspection. Obviously such a requirement introduces undesirable and, to some extent, unnecessary subjectivity into the inference.

Due to the above difficulties and some other problems associated with the identification stage of Box-Jenkins prescription for model selection (see Newbold, 1983), recently time series analysts have considered the Akaike Information Criterion (AIC), the Bayesian Information Criterion (BIC) and the Hannan-Quinn criterion (HQ) as promising tools for model identification.

Akaike (1973) used the Kullback-Leiber information criterion to derive his celebrated criterion:

$$AIC = - \log f(X/\theta) + K \quad (1)$$

where  $f$  is the maximum likelihood vector of the parameter vector  $\theta$  based on realizations (observations), and  $K$  is the number of parameters to be estimated. The criterion suggests that from a pool of competing models, that model should be chosen that minimizes the value of AIC. As can readily be seen from equation (1), AIC consists of two terms, the first term is a measure of the goodness of fit of the model and the second term is a measure of the price that should be paid for increasing the number of parameters. By showing the existence of a trade-off between the fit of the model and the number of parameters to be estimated, Akaike's criterion explicitly formulates the principle of parsimony which advocates the use of the smallest possible number of parameters in the model.

While understanding the overly simplified explanation of AIC given above is crucial to understanding the remainder of the paper, we will not employ AIC for identifying our model. This exclusion is due to AIC's inconsistency in estimating the order of the process which has been pointed out by Shibata (1976) for the AR and Hannan (1980) for the ARMA processes.

The inconsistency of the estimates obtained from the AIC led to the development of a new criterion introduced independently by Akaike (1977), Reissman (1978) and Schwartz (1978) which is commonly known as BIC. BIC, which is strongly consistent, is given by:

$$\text{BIC} = -\log f(X/\theta) + K \log N \quad (2)$$

where  $N$  is the number of independently repeated realizations. Unfortunately, the theoretical framework on which BIC is constructed requires  $N$  to grow to infinity. This requirement reduces the attractiveness of BIC for choosing the "best" model when only a small number of observations are available.

Hannan and Quinn (1979) suggested that the expression  $\log N$  in (2) should be replaced by  $\text{cloglog } N$  ( $c > 1$ ). The rationale behind this suggestion is that while such a change does not affect the consistency of the estimates, it increases the rate of the decrease of the second term as the sample size gets larger. Hence, similar to AIC and BIC, the Hannan and Quinn criterion penalizes an increase in the number of parameters. But as the sample size grows the penalty assigned by the HQ criterion decreases faster than those assigned by AIC or BIC. The HQ criterion is given by

$$HQC = -\log f(X/\theta) + cK \log \log N \quad (3)$$

In equations (1) through (3) K is the number of parameters to be estimated and, hence, for the ARMA models, it is simply  $p + q$ . It can be shown (see Hopwood, McKeown and Newbold, 1984) that the maximum likelihood function can be written as:

$$-\log f(X|\theta) = -\frac{N}{2} \log \hat{\sigma}^2 + (\lambda-1) \sum_{t=1}^N \log X_t$$

where  $\lambda$  is the transformation parameter such that  $\lambda=1$  for a linear model and  $\lambda=0$  for a logarithm model (see below). Hence equation (3) may be rewritten as:

$$HQC = -\frac{N}{2} \log \hat{\sigma}^2 + (\lambda-1) \sum_{t=1}^N \log X_t + c(p+q) \log \log N \quad (4)$$

The Hannon-Quinn criterion, as specified by equation (4) with  $c=2$ , is used below for estimating the dimension of our models.

### III. Model Identification Results

Our data exhibited both a trend and a seasonal component. We transformed the data to a stationary series by first-order differencing and then removed the seasonal component by fourth-order differencing. Hence, our differenced data point will be of the form:

$$Y_t = (1-B)(1-B^4)X_t$$

where B is the backshift operator.

In our search for the "best" linear and log linear models we used the direct derivation of the likelihood function as provided by

Hillmar and Tiao (1979) and employed a program developed at the University of Wisconsin by Tiao et al. (1980) in order to find the maximum value of the likelihood functions. Then we applied the HQ criterion for obtaining the "best" linear and the "best" log linear models. As can be seen from the results summarized in Table 1, in both cases an ARMA(1,0) model was chosen as the most appropriate specification:

$$Y_t = -1.08644 - .487563Y_{t-1} + \epsilon_t \quad (5a)$$

$$\log Y_t = -.005338 - .536118 \log Y_{t-1} + \epsilon_t \quad (5b)$$

Hence, by employing an information criterion we have attained two competing models from which one should be chosen as the better specification.

At this point usually researchers compare the likelihood of the two models and based on this comparison they choose the "better" model. This approach, however, is deficient in some respects. The first, and probably the most significant, deficiency stems from the fact that one model is always chosen even when neither of the models may be significant in describing the phenomenon in question. Furthermore, comparison of likelihoods makes sense only if we compare parameters belonging to the same parameter space (i.e., nested models). In many cases, such as ours, the two models have different parameters in such a way that one model could not be obtained by imposing restrictions on the other model and, hence, they must be regarded as non-nested models. In such situations likelihood comparison should not be regarded as reliable as a means of model selection.



TABLE 1

Modelling the Linear and the Log Linear Forms of ARMA

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<u>Order</u>	<u>Linear Version</u> <u>HOC</u>	<u>Log Linear Version</u> <u>HQC</u>
(0,1)	-261.463	52.675
(0,2)	-256.263	55.317
(0,3)	-251.135	57.974
(0,4)	-240.875	61.683
(0,5)	-226.748	68.570
(1,0)	-275.934	48.358
(1,1)	-256.844	55.097
(1,2)	-247.181	58.066
(1,3)	-234.736	61.490
(1,4)	-234.372	65.007
(2,0)	-264.098	53.469
(2,1)	-252.458	56.489
(2,2)	-242.374	59.269
(2,3)	-238.089	66.465
(3,0)	-258.760	56.196
(3,1)	-250.033	58.865
(3,2)	-236.632	61.620
(4,0)	-253.274	58.988
(4,1)	-231.619	66.091
(5,0)	-246.123	62.579

In recent years a number of tests for comparing non-nested models have been developed (for an excellent survey see McAleer 1982). These tests include the J-test of Davidson and MacKinnon (1981), the JA test of Fisher and McAleer (1981), the PE test of MacKinnon et al. (1983) and a test developed by Bera and McAleer (1982). All the mentioned tests are distributed as  $N(0,1)$  under the null hypothesis in large samples. In non-nested testing the null and the alternative hypotheses are compared to each other. Four outcomes are possible: (1) accept only the null, (2) accept only the alternative, (3) accept both the null and the alternative, (4) reject both the null and the alternative hypothesis.

While the above tests could help us in testing model (5a) against model (5b), they will not be very useful in testing either model (5a) or model (5b) by itself against the data. For example, if one of the tests for non-nested models were performed and the result of the test was "reject both models", such a result would only mean that neither of the specifications is of significance relative to the other model. It does not follow that neither of the models is of significance by itself in explaining the "reality." In other words, the above tests are appropriate for model testing and not for model discrimination.

Since here we are interested in absolute ability of each model in explaining the "reality" and not in its relative ability, and since we are concerned with single series, we propose using the Box-Cox transformation instead of using the available methods for non-nested hypothesis testing. This transformation has been employed for the demand for the money function (Zarembka, 1968 and Spitzer 1976, 1977),

the consumption function (Tsao, 1975), the liquidity trap (White, 1972) and the production function (Berndt and Khaled, 1979). Models (5a) and (5b) can be rewritten as:

$$Y_t = \theta_0 + \theta_1 Y_{t-1} + \varepsilon_t \quad (6a)$$

$$\log Y_t = \beta_0 + \beta_1 \log Y_{t-1} + \varepsilon_t \quad (6b)$$

Employing the Box-Cox transformation yields:

$$Y_t(\lambda) = \beta_0 + \beta_1 Y_{t-1}(\lambda) + \hat{\varepsilon}_t(\lambda) \quad (7)$$

where  $\hat{\varepsilon}$  is a consistent estimator of  $\varepsilon$  and  $\beta_i = \theta_i$ ,  $i = 0, 1$ , when  $\lambda = 1$ . Notice that if  $\lambda = 1$  (6a) and (7) will be identical and if  $\lambda = 0$  (6b) will be identical with (7). Hence a test for the linearity of the model is a test for  $\lambda = 1$  and a test for the log form is a test for  $\lambda = 0$ .

We should also note that if the error terms are normally distributed (as it is assumed in this paper), it may be impossible to have a log model since such a model could result in negative values of  $\log Y_t$ . If, however,  $\bar{Y}_t = \frac{1}{n} \sum_{i=1}^n Y_i$  is several standard deviations greater than zero, then even extreme negative values for the error terms will not yield to negative values for  $\log Y$  and, as result, the assumption of Gaussian error terms is justifiable.

Estimation of  $\lambda$  and its standard deviation can be easily done by using the Box routine in SHAZAM. Our investigation yielded to  $\hat{\lambda} = 0.75$  with the t-ratio of .5468. This result indicates that for 1, 5, or 10 percent level of significance the hypothesis  $\lambda = 1$  can be accepted while

$\lambda = 0$  should be rejected. Therefore the linear form of ARMA(1,0) should be regarded as the "best" model.

#### IV. Forecast Evaluation

We used the following linear and log models for forecasting the out of sample data of 1981 I to 1983 II:

$$X_t = -1.08644 + X_{t-1} + X_{t-4} - X_{t-5} - .487563(X_{t-1} - X_{t-5} - X_{t-2} + X_{t-6}) + \varepsilon_t$$

$$\begin{aligned} \log X_t = & -.005338 + \log X_{t-1} + \log X_{t-4} - \log X_{t-5} \\ & - .536118(\log X_{t-1} - \log X_{t-5} - \log X_{t-2} + \log X_{t-6}) + \varepsilon_t \end{aligned}$$

We also used the linear and the log forms of a simple econometric model which uses the information available on personal income at time  $t$  for predicting the tax revenues of the same time period:

$$X_t = -116142.01 + 6.155I_t + 53270.25Q1$$

(-5.77)      (28.67)      (3.83)

$$+ 168854.56Q2 + 2019.22Q3$$

(12.17)      (.14)

$$\overline{R}^2 = .958 \quad D.W. = 1.729$$

$$\log X_t = -.542 + 1.185 \log I_t + .113Q1$$

(-1.46)      (36.13)      (4.23)

$$+ .369Q2 - .0005Q3$$

(13.86)      (-.02)

$$\overline{R}^2 = .972 \quad D.W. = 1.612$$

where  $X$  is the income tax receipts,  $I$  is personal income and  $Q_i$ ,  $i=1,2,3$ , represents seasonal dummies.<sup>2</sup> The percentage root mean square error (RMSE(%)) is measured by:

$$RMSE(\%) = \frac{1}{\bar{X}} \sqrt{\frac{1}{N} \sum (X_t^s - X_t^a)^2}$$

where  $X_t^s$  is simulated tax receipts, and  $X_t^a$  is actual tax receipts in period  $t$ , and  $\bar{X}$  is the mean of the variable. The results are summarized in Table 2. As can be seen, both the linear and log time series models outperformed the econometric models according to the percentage root mean square error criterion. The linear time series model performed the best, followed by the log linear time series model. Both had substantially smaller root mean square errors than did the econometric models.

## V. Conclusions

In this paper we applied some "objective" standards to the development of a time series model for forecasting state income tax receipts. Using the Hannan-Quinn criterion, we first determined that the linear and log linear versions of the ARMA(1,0) model are preferred over other models. We then used a Box-Cox transformation to select the linear version of the time series model.

When compared with the forecasts from an econometric model, the forecasts obtained from the linear time series model were judged far superior by the percentage root mean square error criterion. This is a significant finding since the econometric model employed more information than did the time series model. In particular, the econometric

TABLE 2

Forecasts and the Percentage Root Mean Square Error  
(RMSE(%)) for Forecasts over the Period 1981.1-1983.2  
(millions \$)

Period	Actual	Linear Econometrics	Log Econometrics	Linear Time Series	Log Time Series
81.1	652.378	728.960	739.907	659.185	661.244
81.2	890.722	865.386	986.048	850.514	851.615
81.3	593.101	718.961	701.531	631.295	619.693
81.4	654.392	727.018	711.955	687.441	599.927
82.1	724.400	774.816	790.958	684.115	678.988
82.2	894.554	905.339	1043.760	959.562	986.778
82.3	625.644	741.550	724.126	629.093	625.273
82.4	580.554	748.290	733.292	671.849	668.769
83.1	711.164	811.870	832.593	701.342	701.390
83.2	881.000	946.314	1103.497	802.534	827.371
	RMSE(%)	.181	.173	.068	.071
	Ranking	4	3	1	2

model depended on personal income for both the in-sample and out-of-sample periods while the time series model used only tax receipts data. Hence, the data requirements for the time series model were substantially less than those of the econometric model. Still, the time series model performed better according to our test.

Forecasting tax receipts with a time series model is not a commonly followed procedure among state tax forecasters. One reason may be the novelty of the approach and the difficulty of identifying an appropriate time series model. The main contribution of this paper is to show that relatively easy to apply techniques are available for time series model selection and that the application of these techniques can lead to a time-series model which outperforms a standard econometric forecasting model.

FOOTNOTES

<sup>1</sup>The Illinois income tax, introduced in 1970, is a flat-rate tax on individual and corporate income applied at rates of 2.5 percent and 4.0 percent respectively. The state of Illinois recently enacted a 20 percent temporary increase in the individual and corporate income taxes to be retroactive to the period 1983 I through 1984 II. Since the temporary tax increase did not impact state tax receipts until 1983 III, the period immediately following our sample data, the temporary tax increase was disregarded in this study.

<sup>2</sup>The numbers in parenthesis are t-ratios,  $\bar{R}^2$  is the adjusted  $R^2$ , and D.W. is the Durbin-Watson statistic.



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